WAVE FLOW OF A THIN LIQUID FILM WITH REDUCED GRAVITATION

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The asymptotic small-parameter method is used to theoretically study steady regimes of flow of thin layers of a viscous liquid over an inclined surface at low film Reynolds numbers under conditions of reduced gravitation.

Under actual conditions at relatively low discharges, the loss of stability of a laminar nonwavy film flow with regard to fairly long-wave perturbations leads to the establishment of flow regimes in which the free surface of the film represents a periodic wave of low amplitude (regime of slightly nonlinear waves) [1]. The smallness of the "long-waviness" parameter – the ratio of the unperturbed thickness of the film to the wavelength of the perturbation – and the relative smallness of the film Reynolds numbers makes it possible to use the small parameter method to study this regime. Thus, the following nonlinear evolutional equation was obtained in [2] in a first approximation to describe the wave flow of a thin liquid film:

$$\frac{\partial\varphi}{\partial t}+\frac{\partial}{\partial x}\left[\left(1+\frac{1}{3}T\frac{\partial^3\varphi}{\partial x^3}\right)\eta^3\right]-\varepsilon\frac{\partial}{\partial x}\left[\mathrm{tg}\,\alpha\frac{\partial\varphi}{\partial x}\eta^3+\right.$$
$$\left.+\mathrm{Re}\left(\frac{5}{24}V_2+\frac{3}{20}V_3\eta+\frac{7}{60}V_4\eta^2+\frac{2}{21}V_5\eta^3\right)\eta^4\right]=0. \quad (1)$$

Here, we have introduced the following dimensionless variables (the primes denote the corresponding dimensional variables):

$$t=\frac{u_0}{\lambda}t',\ x=x'/\lambda,\ y=\frac{y'}{h_0},\ \begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\frac{1}{u_0}\begin{Bmatrix}v_x'\\ v_y'\end{Bmatrix},$$
$$p=\frac{\mathrm{Re}}{\rho u_0^2}p',\ \varphi=\frac{h-h_0}{h_0}, \quad (2)$$

and the parameters

$$\mathrm{Re}=\frac{u_0h_0}{\nu},\ T=\frac{3\varepsilon^3\,\mathrm{We}}{\cos\alpha},\ \mathrm{We}=\frac{\sigma}{\rho g h_0^2},\ \varepsilon=\frac{h_0}{\lambda},$$
$$u_0=\left(\frac{\cos\alpha}{3}\frac{g}{\nu}\right)^{1/3}Q^{2/3},\ h_0=\left(\frac{3}{\cos\alpha}\frac{\nu Q}{g}\right)^{1/3},$$

where u_0 and h_0 are the mean velocity and thickness of the film in the nonwavy regime; α is the angle of inclination to the vertical. We also introduced the function φ and its derivatives:

$$V_2=\frac{\partial F}{\partial t},\ V_3=\frac{1}{2}\left(H\frac{dF}{dx}-T\frac{\partial^4\varphi}{\partial t\partial x^3}\right),$$
$$V_4=-\frac{1}{3}TG,\ V_5=\frac{1}{12\eta}TG,\ \eta=1+\varphi,$$
$$F=3\varphi+T\frac{\partial^3\varphi}{\partial x^3}\eta,\ H=\left(3+T\frac{\partial^3\varphi}{\partial x^3}\right)\eta,\ G=H\frac{\partial^4\varphi}{\partial x^4}.$$

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Equation (1) was obtained from the basic system of equations of motion in an approximation of the thin liquid layer and the corresponding boundary conditions with $\varepsilon \tan\alpha \lessgtr 1$ (the angles of inclination of the surface to the vertical may be fairly close to $\pi/2$) and $T \lessgtr 1$ (the film number of the investigated liquid $\mathrm{Fi} = \sigma^3/\rho^3 g\nu^4$ is fairly high). The latter condition allows us to study either the flow of liquids with a high surface tension and low viscosity or the flow of any liquid under conditions of reduced gravitation. The authors of [2] studied steady flows of a thin liquid film with $T \sim \varepsilon$. However, the range of validity of these results is very limited as a result of the requirement that $\mathrm{Re} \sim 1$. It should be noted that, given low values of the force of gravity, the case $\mathrm{Re} \sim 1$ is the most realistic.

We will limit ourselves below to studying slightly nonlinear waves ($\varphi \ll 1$) under reduced gravitation ($T \sim 1$) with small angles of inclination of the flat substrate to the vertical ($\tan\alpha \lessgtr 1$). Replacing the derivative $\partial/\partial t$ by $-3\partial/\partial x - {}^1/_3 \partial^4/\partial x^4$ in the first-order terms of the small parameter in (1) and at once considering the relation between the small parameters $\varepsilon\mathrm{Re}$ and φ, such as $\varepsilon\mathrm{Re} \lessgtr \varphi^2$, we obtain ($\varepsilon = 1$, with the selection of an arbitrary scale equal to h_0)

$$\frac{\partial\varphi}{\partial t} + 3(1+\varphi)^2\frac{\partial\varphi}{\partial x} + A\frac{\partial^2\varphi}{\partial x^2} + B\frac{\partial^4\varphi}{\partial x^4} + C\frac{\partial^5\varphi}{\partial x^5} + D\frac{\partial^8\varphi}{\partial x^8} + 3B\frac{\partial}{\partial x}\left[(1+\varphi)\varphi\frac{\partial^3\varphi}{\partial x^3}\right] = 0, \quad (3)$$

$$A = \frac{6}{5}\mathrm{Re} - \mathrm{tg}\,\alpha,\quad B = \frac{\mathrm{We}}{\cos\alpha},\quad C = \frac{10}{7}\frac{\mathrm{We\,Re}}{\cos\alpha},\quad D = \frac{2}{5}\frac{\mathrm{We^2\,Re}}{\cos^2\alpha}.$$

We will represent φ in the form:

$$\varphi = \sum_{n=-\infty}^{\infty}\Phi_n \exp(in(\omega t - kx)),\quad \omega = \Omega - i\gamma,\quad k = 2\pi\frac{1}{L}. \quad (4)$$

Here, k, Ω, and γ are real, and it follows from the realness of φ that $\Phi_{-n} = \Phi_n^*$, where the asterisk denotes complex conjugation. For slightly nonlinear waves, $\Phi_n \sim q^{n/2}$ [3], where $q = \Phi_{-1}\Phi_1 = |\Phi_1|^2$ is the square of the wave of the first harmonic. Thus, considering that Eq. (3) was obtained with an accuracy to within the terms $\varphi^3 \sim q^{3/2}$, only terms of the order q can remain in the expansion (4), i.e., only harmonics with the numbers $|n| \le 2$. Inserting (4) into (3), we have the following for n = 1 and n = 2, respectively:

$$[i(\omega - 3k - Ck^5) - Ak^2 + Bk^4 + Dk^8]\Phi_1 - 3(ik - Bk^4)\Phi_1^2\Phi_{-1} - 6ik(\Phi_0\Phi_1 + \Phi_2\Phi_{-1}) + 3Bk^4(\Phi_0\Phi_1 + 7\Phi_2\Phi_{-1}) = 0,$$

$$[i(\omega - 3k - 16Ck^5) - 2Ak^2 + 8Bk^4 + 128Dk^8]\Phi_2 - 3(ik - Bk^4)\Phi_1^2 = 0.$$

From which it follows that

$$\Phi_2 = \frac{3(ik - Bk^4)}{i(\omega - 3k - 16Ck^5) - 3Ak^2 + 8Bk^4 + 128Dk^8},$$

$$i(\omega - 3k - Ck^5) - Ak^2 + Bk^4 + Dk^8 = q\Big\{3ik - 3Bk^4 + 3\Phi_0(2ik - Bk^4) + \frac{9(ik - Bk^4)(2ik - 7Bk^4)}{i(\omega - 3k - 16Ck^5) - 2Ak^2 + 8Bk^4 + 128Dk^8}\Big\}. \quad (5)$$

The quantity Φ_0 is determined by the condition whereby the dimensionless discharge of the liquid in the perturbed film must be equal to its prescribed value [to unity in the variables (2)]. As in [2], $\Phi_0 = -2q$.

Equation (5) allows us to determine ω (and thus Ω and γ) as a function of the wave number k and the square of the amplitude q. Considering the smallness of q, we obtain

$$\Omega = \Omega_1(k) + q\Omega_2(k),\quad \gamma = \Gamma_1(k) + q\Gamma_2(k).$$

Thus,

$$\Omega_1 = (3 + Ck^4)k,\quad \Gamma_1 = (A - Bk^2 - Dk^6)k^2,$$

$$\Omega_2 = -9k + \frac{27k^3[5C(7B^2k^6 - 2) + 3B(A - 7Bk^2 - 127Dk^6)]}{225C^2k^6 + (A - 7Bk^2 - 127Dk^6)^2},$$

$$\Gamma_2 = 3Bk^4 - \frac{9\,[(7B^2k^6-2)(A-7Bk^2-127Dk^6)-135BCk^6}{225C^2k^6+(A-7Bk^2-127Dk^6)^2}\,.$$

Since a steady regime corresponds to zero increment in the growth of the oscillations γ and this value corresponds to the maximum value of γ regarded as a function of the wave number k [3], we obtain the following two equations:

$$\Gamma_1(k)+q\Gamma_2(k)=0 \quad \text{and} \quad \frac{d\Gamma_1}{dk}+q\frac{d\Gamma_2}{dk}=0$$

determining k and q. From here, with a prescribed accuracy,

$$q=-\frac{\Gamma_1(k_m)}{\Gamma_2(k_m)}\,,\quad k=\frac{\Gamma_1(k_m)}{\Gamma_2(k_m)}\frac{d\Gamma_2}{dk_m}\left(\frac{d^2\Gamma_1}{dk_m^2}\right)^{-1}+k_m,$$

$$c=\frac{\Omega}{k}=\frac{\Omega_1(k_m)}{k_m}+\left[\frac{d\Gamma_2}{dk_m}\left(k_m\frac{d^2\Gamma_1}{dk_m^2}\right)^{-1}\left(\frac{d\Omega_1}{dk_m}-\frac{\Omega_1(k_m)}{k_m}\right)-\Omega_2(k_m)\right]\frac{\Gamma_1(k_m)}{\Gamma_2(k_m)}\,, \tag{6}$$

where k_m is the wave number for waves of maximum growth (the wave number for which the linear increment of the oscillation build-up is maximal).

Equations (6) allow us to obtain analytical expressions for k, q, and c. However, in view of the awkwardness of these expressions, it would probably be more sensible to construct their approximations. Thus, for $\mathrm{Fi}^{0.1}$ = 12.5-15 and $\mathrm{Fi}^{0.1}$ = 25-30 (for example, water at g = 1.0 m/cm^2 and g = 0.001 m/cm^2), with α = 0 we have, respectively:

$$k\,\mathrm{Re}=0.9715\left(\frac{\mathrm{Re}^3}{\mathrm{We}}\right)^{1/2}-3.5396\left(\frac{\mathrm{Re}^3}{\mathrm{We}}\right)^{3/2}+96.057\left(\frac{\mathrm{Re}^3}{\mathrm{We}}\right)^{5/2},$$

$$q=0.06\frac{\mathrm{Re}^3}{\mathrm{We}}-0.6743\frac{\mathrm{Re}^6}{\mathrm{We}^2}+35.2759\frac{\mathrm{Re}^9}{\mathrm{We}^3}\,, \tag{7}$$

$$c=3+0.0194\frac{\mathrm{Re}^3}{\mathrm{We}}-77.2376\frac{\mathrm{Re}^6}{\mathrm{We}^2}+246.3718\frac{\mathrm{Re}^9}{\mathrm{We}^3}\,.$$

$$k\,\mathrm{Re}=0.7698\left(\frac{\mathrm{Re}^3}{\mathrm{We}}\right)^{1/2}-14.8448\left(\frac{\mathrm{Re}^3}{\mathrm{We}}\right)^{3/2}+184.4019\left(\frac{\mathrm{Re}^3}{\mathrm{We}}\right)^{5/2}.$$

$$q=0.06\frac{\mathrm{Re}^3}{\mathrm{We}}+1.324\frac{\mathrm{Re}^6}{\mathrm{We}^2}+11.9969\frac{\mathrm{Re}^9}{\mathrm{We}^3}\,.$$

$$c=3-7.9926\frac{\mathrm{Re}^3}{\mathrm{We}}+1013.207\frac{\mathrm{Re}^6}{\mathrm{We}^2}-12791.8\frac{\mathrm{Re}^9}{\mathrm{We}^3}\,. \tag{8}$$

$$\mathrm{Re}^3/\mathrm{We}=3^{2/3}(\mathrm{Re}/\mathrm{Fi}^{1/11})^{11/3}.$$

Equations (7) and (8) are valid for small Reynolds numbers [Eqs. (7) for Re ≤ 5, Eqs. (8) for Re ≤ 8]. The error of the approximating formulas is no greater than 10%.

Thus, a reduction in the force of gravity leads first of all to a certain expansion of the region of stability of laminar flow with a constant film thickness (the instability shifts in the direction of greater long-wave perturbations), and the critical film Reynolds number remains the same as under actual conditions (with α = 0, Re_{cr} = 0); second, the high film numbers Fi hinder the appearance of waves which are close to harmonic, and these waves are stabilized.

NOTATION

A, B, C, D, functions introduced in (3); c, dimensionless phase velocity; F, G, H, functions introduced in (1); g, acceleration due to gravity; h, h_0, film thickness in the wavy and nonwavy regimes; k, wave number; k_m, wave number for waves of maximum growth; L, dimensionless wavelength; p, p', dimensionless and dimensional pressure; q, square of the amplitude of the first harmonic of the wave; T, parameter in (1); t, t', dimensionless and dimensional time; u_0, mean velocity in the nonwavy regime; V_i, functions introduced in (1); $\vec{v}$, $\vec{v}'$, dimensionless and dimensional velocity; x, x', y, y', dimensionless and dimensional longitudinal and transverse coordinates; α, angle of inclination of the flat substrate to the vertical; γ, increment of oscillation build-up; ε, "long-waviness" parameter; η, dimensionless film thickness; λ, linear longitudinal scale; ν, kinematic viscosity; ρ, density of the liquid; σ, surface tension; φ, dimensionless amplitude of the wave; Ω, frequency of the wave; ω, complex frequency of the wave; Fi, film number; Re, We, Reynolds and Weber numbers; *, complex conjugation.

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PRESSURE PULSATION MECHANISM IN A NONUNIFORM FLUIDIZED BED

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A model is proposed for pressure oscillations in a fluidized bed. A formula is obtained for the oscillation frequency and calculated values are compared with experimental data.

Three types of pressure pulsations are evidently possible in a nonuniform fluidized bed, which is an oscillatory system. The first type is connected is the washing of the pressure measurement device by a bubble in the bed [1, 2]. The second type is connected with the free oscillations in the system comprised of the bed and the gas inlet. These oscillations determine the state of the bed when the resistance of the gas distributor is low and the pregrate volume is high [3, 4]. The third type of pulsation, which predominates in ordinary nonuniform beds, is connected with the natural frequency of the gravitational oscillations of the bed. Todes [5] proposed a formula to evaluate the frequency of these oscillations $\nu = g^{0.5}/(2\pi H^{0.5})$. It correctly reflects the dependence of ν on H, but the values of the pressure pulsation frequency it gives are only half as great as the experimental values. For near-uniform fluidization conditions, i.e., for shallow beds or low fluidization velocities, the following relation was obtained [6]:

$$\nu = \sqrt{g(2-\varepsilon)}/(2\pi\sqrt{H\varepsilon}). \qquad (1)$$

This formula gives the best agreement with the experimental data at $\varepsilon = 0.4$–0.5. However, the assumption of a synchronous change in porosity over the entire height of the bed which was made in deriving the formula is inconsistent with physical representations of particle fluidization, and the authors themselves note that the change in porosity is actually propagated in the form of a wave from the grate to the top boundary of the bed. The present work studies pressure pulsations in the volume of a large apparatus in the regime of intensive bubble fluidization.

Observations show that at the moment a gas bubble escapes, the column of particles underneath the bubble rises. At the same time, the particles in the adjacent regions descend and circulation loops are formed (Fig. 1). To calculate the natural frequency of the bed oscillation, we will simplify the actual situation and represent it in the form of oscillations of an ideal liquid in a U-shaped tube (Fig. 1). The equation describing the oscilla-

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